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LINEAR PROGRAMMING TECHNIQUES APPLIED TO RESEARCH PLANNING

by

Leslie Stoessl
Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
OPERATIONS RESEARCH

United States Naval Postgraduate School
Monterey, California

1964

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U. S. Naval Postgraduate School
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LINEAR PROGRAMMING TECHNIQUES
APPLIED TO RESEARCH PLANNING

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Leslie Stoessl

This work is accepted as fulfilling
the thesis requirements for the degree of
MASTER OF SCIENCE
IN
OPERATIONS RESEARCH
from the
United States Naval Postgraduate School

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This thesis is a result of a work performed during a summer assignment at the Office of Naval Research. The topic was inspired by Dr. F. D. Rigby's paper *Activity Analysis of Research Planning in ONR.*

I wish to thank all the members of the Naval Analysis Group who assisted me and particularly Dr. Marshall C. Yovits and Captain J. F. Gustaferro, USN, for their leadership and direction during this period. I should also thank Dr. Fred Rigby, Commander Ernest Stavely and especially Mr. Herman I. Shaller for their technical assistance, advice and encouragement. Mr. Shaller's report, *An Exploratory in Research Methodology, in many* instances provided the notation and mathematical definitions used herein.

I would also like to express my sincere thanks to Professor Rex H. Shudde for his technical advice while acting as my thesis advisor.

ABSTRACT

Linear programming has been used to maximize allocation problems in many fields. This technique will be applied to the problem of planning research with optimal allocation of limited resources. A model is constructed with detailed information concerning necessary inputs and resultant outputs, amplified by sample problems. A sensitivity analysis and interpretation of all results is included. The model has been constructed to be of primary interest to the Office of Naval Research but can be adjusted to the needs of other groups engaging in research.

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INTRODUCTION

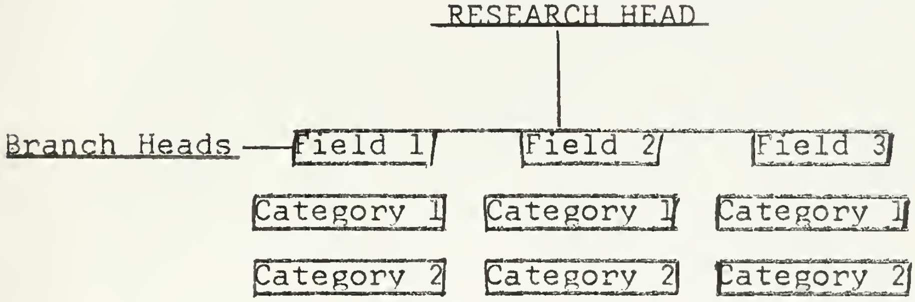
In recent years, the increased emphasis on research and development, by both private and governmental organizations, has raised the questions "Can research be planned?" and "If so, can it be planned optimally?" This report concerns the second question, the problem of the optimal allocation of funds for the accomplishment of a research objective. The technique proposed in this thesis to answer the second question is linear programming.

Linear programming was first suggested as a technique to explore the area of research planning by Dr. Fred Rigby of the Office of Naval Research. His paper "Activity Analysis of Research Planning in ONR", written in 1963, proposed the feasibility of constructing a model predicated on the research effort of ONR and utilizing linear programming.

Prior to the paper by Dr. Rigby, the Office of Naval Research had completed a study of research planning which was called the "Ad Hoc Research Planning Committee Report." This study effort was headed by Captain J. F. Gustaferro, USN. An outgrowth of the study was a report by Herman I. Shaller, also a member of the Ad Hoc Committee; his report is titled "An Exploratory Study in Research Planning Methodology".

This thesis utilizes the information in the above reports and is an attempt to implement the suggestion made by Dr. Rigby to utilize linear programming for the investigation of research planning.

For the purpose of the thesis, an organization diagram of ONR, Appendix 4, has been simplified into the following form:



A total budget is received by the research head, and is passed on to the department or branch heads. These individuals, each heading a particular scientific field, such as biology or physics, must then decide how to allocate available funds among the various activities comprising their particular departments. The smallest sub-division of a particular department engaging in research is called a category. Examples of categories in the Electronics Department of ONR may be found in Appendix 3, page (72). This thesis assumes that the department heads will divide the total budget among the categories rather than by making an initial allocation to the departments. The problem,

therefore, is that of dividing the budget among the categories to maximize the research effort.

To maximize the research effort of the organization, the particular mission of that organization must be examined. The mission of ONR is:[1]

> to encourage, promote, plan, initiate, and coordinate naval research to provide for the maintenance of future naval power and the preservation of national security; to conduct naval research in augmentation of and in conjunction with research and development conducted by the respective bureaus and other agencies and officers of the Department of the Navy; to supervise, administer and control all activities within or on behalf of the Department of the Navy relating to patents, inventions, trademarks, copyrights, royalty payments and matters connected therewith; to represent the Department of the Navy in dealings of Navy-wide interest on research matters with other government agencies, corporations, educational and scientific institutions, and other organizations and individuals concerned with scientific research; to survey the world-wide findings, trends, potentialities and achievements in research and development, keep the ASN (R and D) and the Chief of Naval Operations advised thereon, and disseminate such information as appropriate to interested bureas and offices within the Department of the Navy, and to other Governmental or private agencies.

For the purpose of the linear programming model, this mission must be described by a particular distinct set of objectives which must be internally independent. ONR calls these objectives attributes. Clearly the mission of an organization may be described by more than one set of attributes or objectives. Each particular set of attributes

---

[1] Office of Naval Research Organization Manual, ONR Inst. 5420.1E, Aug. 1961

which describe the mission is called a classification system. Examples of attributes in a classification system might be scientific value, military value, and technological value. Examples of other classification systems used in the Navy are shown in Appendix 3, page (73). Therefore, attributes are organizational objectives.

Inter-relations and dependencies exist between Navy bureaus and the research activity of ONR. An example of such a situation may be described in this manner: Assume that scientific value, military value, and technological value comprise one classification system, and that anti-submarine warfare, air warfare, and surface warfare are another, both describing ONR's research effort. Increased research activity in underwater sound transmission will not only affect the military attribute of the first classification system, but the anti-submarine warfare attribute of the second classification system as well. The model is developed to recognize these interdependencies and still achieve an optimal resource allocation with consideration given to other classification systems. Therefore, category support in one system will affect results in other classification systems.

To apply linear programming techniques to the problem of research optimization involved translating the following

linear programming concepts: maximize $c^T X$ subject to constraint equations $AX \geq b$ and $X \geq 0$, where A denotes the technology matrix, b denotes the requirement vector, c^T denotes the cost coefficients and X denotes the unknowns.

Symbology and definitions used in the above translation were originally formulated by H. Shaller [4]. His paper outlined a method of displaying an existing research program in matrix form. This report attempts to find a method of maximizing the results of a research program as well as finding the most effective way to change the program when new conditions arise.

In the proposed model, the unknowns to be determined are the amounts of funds to be assigned to a category to optimize program effectiveness. These unknowns are denoted by X. c^T denotes a set of coefficients reporting the effectiveness of a category to the mission of an organization per research dollar invested in that category. The term category effectiveness refers to the amount of progress made in achieving the mission of the organization per dollar input to that category. Therefore, $c^T X$ is the objective function to be maximized.

To utilize the experience of the research manager, constraint equations are formulated so that the requirements vector b is composed of the set of all category funding levels. Each category has two funding levels.

These funding levels are the lower and upper bounds on the amount of funds the research manager desires to spend for a particular category. A lower funding level may be computed by determining the minimum amount of funds a category must receive if any research is to be accomplished. An upper funding level can be chosen from a history of past funding to a particular category or by the research manager evaluating a category's potential contribution to his mission.

Because of formulating the constraint equations in this manner, each element of the technology matrix $\underline{A}$ takes on a value, -1, 1, or 0. Therefore, the objective function is maximized by selecting levels of category funding between a lower and upper funding level previously determined by the research manager. Further explanation and examples of these concepts are found in the sections titled Model Inputs, page (19).

The questions to be examined in this thesis were originally discussed by the author with personnel of ONR during a summer assignment there in 1963.

These questions were also posed in the Ad Hoc report previously completed. They are:

1. What is the effect of a budget alteration on the present research program? Can all categories continue to be supported? What activities must be curtailed or reduced,

and by how much, to achieve optimum program effectiveness within this new budget?

2. If a new category is added to the research program, how must I modify the existing program. What is the optimal way for this change to be accomplished?

3. What is the effect of a change of emphasis on the program? How should a reallocation of funds to accommodate this change of emphasis be made?

These questions are not necessarily exhaustive. New questions may occur to the reader which can be answered by applying the linear programming technique.

To answer these questions six problems were arbitrarily developed. These problems were solved with the aid of the CDC 1604 computer of the Naval Postgraduate School, Monterey, California. Because of the tedium involved in applying the linear programming technique to a large problem and the difficulty in inverting large matrices, computers are essential in applying this method. Because computers must be used in the solution of the problem, care must be exercised to insure that the size of the problem formulated does not become insolvable due to limitations of existing computers. The problems would be unmanageable if too many constraint equations were formulated due to a large number of categories in use. Grouping of categories might then be necessary.

The data used in the sample problems of the thesis are arbitrary and have no particular significance. The first two problems are maximizations of a program's effectiveness when two different classification systems, identified as A and B are used to identify the same research program. The results of these problems indicate the allocation of funds which optimize program effectiveness. Problem three combines the A and B system by arbitrarily adding their objective functions. The results are not of great interest unless a research organization can operate in this manner. Problem four examines the effect of perturbing the requirements vector. Problem five, the most interesting, and perhaps most useful, examines the effect of optimizing the research effort of a particular classification system when subject to the constraining influence of another classification system. This problem approaches realism to a greater degree than do problems one and two. Problem six explores the effects of a budget cut on the research program. A sensitivity analysis is conducted on problem one to evaluate the sensitivity of the solution obtained with respect to variations in the input parameters. Detailed results were obtained for all six problems. From these results the following general conclusions can be drawn:

1. The linear programming model can be of great assistance in planning a research program providing a means of evaluating the reliability of the input data exists.
2. To assist in formulating the input data, further study is recommended into the problem of quantitatively measuring the output of a research program.

THE MODEL

ASSUMPTIONS

ASSUMPTIONS

Primary Assumptions

For model construction purposes, it is assumed that the total research domain can be divided into subdomains of manageable size, that is, the number of categories does not result in too many equations for solution by present computer techniques. Homogeneity in the branches and categories must exist in order to establish independence among the linear equations to be formed. Homogeneity can be approximated through proper category definition. For our model, homogeneity and manageability of the branches and categories are assumed to be present.

The program managers, through judicious application of management techniques, achieve within-activity optimization.[1] This implies that for a given resource input to a category, the research manager will so use his resources that the quantity and quality of research output is optimized. A graph of the relationship between resource input and research output[2] follows:

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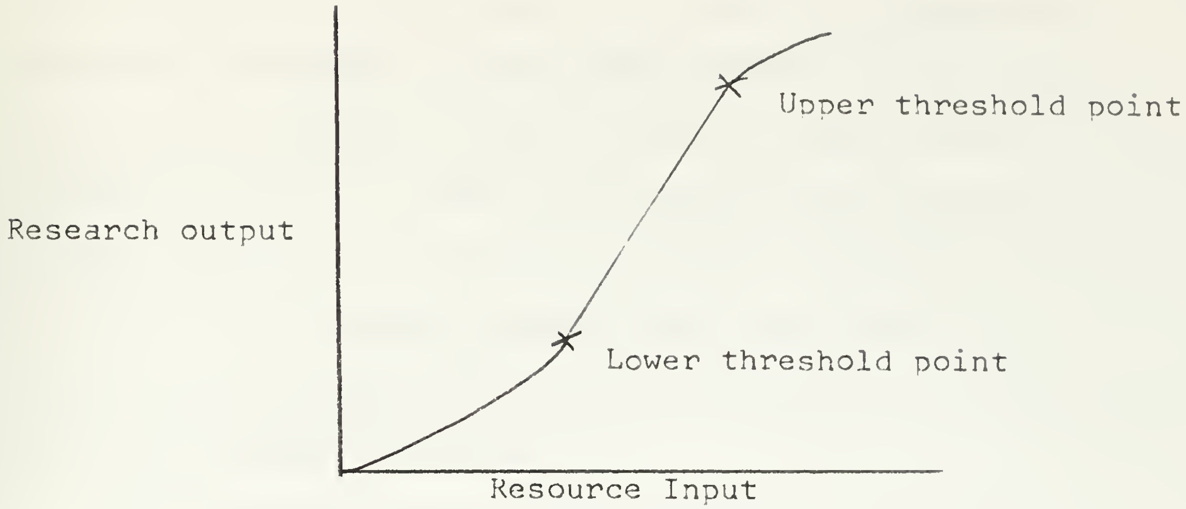
[1] Rigby, F. D. Activity Analysis of Research Planning in ONR, 1963.

[2] Ad Hoc Research Planning Committee at the Office of Naval Research.



Graph 1

The lower portion of the curve exemplifies that situation where a research project has just begun. In this range, output in relation to input is low. Possible reasons for this may be that the group has not "jelled," proper equipment is not yet on hand, or the problems themselves are not completely formulated. The major portion of the curve is approximately linear with an unknown slope. Here, the assumption is made that a large number of workers adequately financed will progress in a linear manner toward solution of a problem. Research teams are now organized, goals have been set, and all the inputs are now present for problem resolution. This area is the most prevalent one in today's research organizations. The upper portion of the curve constitutes a region of diminishing returns for resource

input. Such a condition may occur when severe scientific roadblocks, requiring a major breakthrough to clear, are present. An example of such a situation might be the problem of finding a light but effective shielding for nuclear reactors to be used in automotive transportation. A region of diminishing returns might also occur if adequate experimental equipment or sufficient new scientific personnel were not available.

Two points of interest, therefore, are the lower and upper activity thresholds. An ability to estimate the lower threshold point would enable the research manager to evaluate when a large output return would be imminent for small resource input. Knowledge of the upper threshold point would enable him to partially shift his resources to another category when diminishing output was impending. An evaluation of these threshold points will not be attempted in this report.

Studies are presently being conducted to examine the feasibility of measuring research output. As yet no such measuring device for ONR has been found, thereby necessitating another major assumption. The efficiency of a particular category with respect to an attribute is a necessary model input. This efficiency is defined as research output divided by resource input. The sum of a category's efficiency to all the attributes must equal one.

Consequently, estimates of category efficiency are entirely subjective. Therefore, we must assume that experience will enable the project manager to make a reasonable estimation of this quantity. This subjective estimation is a model weakness.

Secondary Assumptions

It is logical to assume that a certain level of resource input per category is essential if a meaningful amount of research is to result. The minimum amount of financial support a category must receive if progress is expected must therefore be established. It is assumed that meaningful research can be accomplished with a minimum of one unit of research [3] where a unit of research is defined as the research input necessary to support the average research scientist for a given time interval. For model purposes, the time interval will be one year. A necessary lower bound on category financing, therefore, is the cost of maintaining one unit of research in a particular category.

The determination of what constitutes a "research scientist" and the evaluation of his work is entirely subjective. Junior researchers may be weighted less

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[3] Shaller, H. I. An Exploratory Study in Research Planning Methodology, ONR Report ACR/NAR-27, Sept. 1963.

heavily than senior researchers. Greater emphasis may be placed upon the work of a particularly outstanding individual. By appropriate adjustment, therefore, the unit of research will be used as the least common denominator for all quantitative arguments.

For purposes of simplicity, the proposed model will not include those conditions where financing is done over a period greater than one year. If these constraints are desired for practical application purposes, they may be formed as yearly ratios and included in the array of constraint equations.

CONSTRAINTS

CONSTRAINTS

Optimization of the research effort is defined as that set of resource allocations which achieves the greatest effectiveness under a given set of constraining conditions. Constraining conditions are essentially those bounding factors which the research manager desires to be considered in the overall solution to his problem. The restrictions to be discussed are budget constraints, balance constraints, and threshold constraints.

Budget constraints consist of those limits superimposed on the research program by monetary considerations. These budgetary allocations may be presented in a number of forms; as a single ceiling under which the research manager must conduct his entire program; as a series of budget ceilings placed on individual branches or categories. In the former case, there exists essentially one bound, while the latter example contains constraints equaling the number of categories individually budgeted.

A threshold constraint might be defined as the minimum resource allocation to a category necessary to conduct meaningful research. It has been previously assumed that meaningful research can be accomplished with a minimum of one research unit, therefore the threshold constraint per category is the cost of maintaining a research unit in a

particular category. The expression "meaningful research" is subjective. As there does not exist a measuring device for research output, the "one unit of research" criteria was chosen essentially by experience. Therefore, the cost criteria for "meaningful research" can be chosen arbitrarily.

MODEL INPUTS

MODEL INPUTS

Five items will be discussed as inputs to the model, of which two are constants, two are subjective variables, and one is a subjective constant.

CONSTANTS

A total and fixed budget over a yearly interval must be specified. This budget may later be influenced by the conclusions drawn from model results, but a fixed amount of dollars for research purposes must initially be determined. The total budget for ONR is determined within the Defense Department. X will denote the total budget.

As previously noted, a unit of research is defined as the resource input necessary to support the average research scientist for one year. The cost of a unit of research will naturally vary widely among different categories, depending upon the area of investigation. A research scientist engaged in subatomic particle study might require the use of highly sophisticated equipment while a mathematician investigating a theorem might need only pencil and paper. Cost per research unit in a particular category might also include such items as floor space, supporting personnel, salaries, and overhead. The total cost per research unit in the ith category will be denoted by $K(C_i)$.

SUBJECTIVE VARIABLES

The overall mission of an organization is clearly specified in terms of such attributes as maximization of profits, maximization of military might, or maximization of research progress. After breaking down a specific organizational mission into its attributes, a decision must be made as to what emphasis is to be placed on each attribute. The emphasis factors will be in percentage form, with their sum equal to one hundred percent. The decision as to what contribution an attribute is to make to the overall mission will originate from the upper levels of an organization. The direction a research organization will take is now determined. $C(A_j)$ will denote the contribution of the j^{th}attribute to the overall mission.

Another model input is the approximate emphasis that is to be placed on each category. This factor may be determined from study of the history of the organization, from an estimate of the importance of a particular category to the overall mission, or from empirical observation. This decision would usually be made at a lower organizational level--in ONR by the department heads. These estimates are to be assigned in bracket form. For example, 10 to 15 percent might be assigned to category one, and 30 to 40 percent to category two. Care must be taken to

insure that the sum of the lower estimates is less than 100 percent and the upper estimates sum to greater than 100 percent. $W(C_i)$ will denote emphasis to be placed on the i^{th} category.

SUBJECTIVE CONSTANT

The efficiency or progress a category contributes to an attribute has previously been discussed in the section titled ASSUMPTIONS. The project manager must decide what contribution his category makes to a particular attribute. The sum of these percentages must equal one hundred. The degree of difficulty in making such a decision will naturally vary with each category. A category such as propaganda obviously has a large preponderance of military rather than scientific value. A project manager concerned with the propaganda category may decide therefore that 15 percent of this category contributes to attribute one, 60 percent to attribute two, and 25 percent to attribute three. It is recognized that this subjective estimate may at times be very difficult to make. Progress contributed by the ith category to the jth attribute will be denoted by P_{ij}.

MODEL INPUTS

1. X - total budget
2. $C(A_j)$ - Contribution of the jth attribute to the overall mission.
3. $W(C_i)$ - Emphasis placed on the ith category
4. P_{ij} - Progress contributed by the ith category relative to the jth attribute per dollar.
5. $K(C_i)$ - Total cost per unit of research in the ith category.
   $K(C_i)$ is expressed as:

   $$K(C_i) = a_i z_i + b_i v_i + c_i w_i + d_i y_i \ldots$$

where:

1. $a_i = \frac{\text{cost}}{\text{scientist year}}$
2. $z_i = \frac{\text{scientist year}}{\text{unit of research}}$
3. $b_i = \frac{\text{cost}}{\text{unit of personnel}}$
4. $v_i = \frac{\text{unit of personnel}}{\text{unit of research}}$
5. $c_i = \frac{\text{cost}}{\text{sq.ft. of office space, lab space...}}$
6. $w_i = \frac{\text{sq.ft. of space required}}{\text{unit of research}}$
7. $d_i = \frac{\text{cost}}{\text{unit of equipment required}}$
8. $y_i = \frac{\text{piece of equipment required}}{\text{unit of research}}$
9. Scientist year -- unit of research

MODEL OUTPUTS

MODEL OUTPUTS

Direct Output

After consideration of the inputs previously discussed, the model indicates the amount of resources to be allocated to each category. The resource allocation to the ith category will be noted x_i.

Computed Output

Upon arriving at a set of optimal resource allocations, certain additional quantities may be of interest. These values will be displayed in matrix arrays as formulated by H. Shaller in An Exploratory Study in Research Planning Methodology. [4]

COMPUTED MODEL OUTPUTS

1. u_i = Number of research units in the ith category.
2. u_{ij} = Number of research units in the ith category corresponding to the jth attribute.
3. E_{ij} = Effectiveness of the ith category relative to the jth attribute to the overall mission.
4. E_{ia} = Effectiveness of the ith category relative to all the attributes per research unit.
5. E_i' = Effectiveness of the ith category relative to all the attributes per research unit.
6. $\bar{E}$ = Total effectiveness of the research program.
7. $E(B_j)$=Emphasis resulting on the jth goal of the B classification system.

NOTATION

1. C_i = The i^{th} category.

2. A_j = The j^{th} attribute.

3. $C(A_j)$ = Contribution of the j^{th} attribute to the overall mission.

4. x_i = Resource allocation to the i^{th} category.

5. X = Total resources.

6. $K(C_i)$ = Cost per research unit in the i^{th} category.

7. u_i = Number of research units in the i^{th} category.

8. u_{ij} = Number of research units in the i^{th} category corresponding the the j^{th} attribute.

9. U = Total number of research units accomplished.

10. p_{ij} = Progress contributed by the i^{th} category relative to the j^{th} attribute per dollar

11. E_{ij} = Effectiveness of the i^{th} category relative to the j^{th} attribute.

12. E_{ia} = Effectiveness of the i^{th} category relative to all the attributes.

13. $\bar{E}$ = Total effectiveness of the research program.

14. $W(C_i)$ = Emphasis placed on the i^{th} category.

15. E_i' = Effectiveness of the i^{th} category relative to all the attributes per research unit.

16. $E(B_j)$ = Emphasis resulting on the j^{th} goal.

$i = (1, 2, \ldots.m)$, $j = (1, 2, \ldots.n)$

MATHEMATICAL DEFINITIONS AND BASIC EQUATIONS

MODEL INPUT RESTRAINTS

1. $\sum_{j=1}^{n} C(A_j)=1$
2. $\sum_{i=1}^{m} W(C_i)=1$
3. $\sum_{j=1}^{n} D_{ij} =1$

MODEL OUTPUT DEFINITIONS

1. $\sum_{i=1}^{m} x_i = X$
2. $u_i = \frac{x_i}{K(C_i)}$
3. $E_{ij} = D_{ij}\, C(A_j)$
4. $E_{ia} = u_i E_i'$
5. $E_i' = \sum_{j=1}^{n} E_{ij}$
6. $\bar{E} = \sum_{i=1}^{m} E_{ia}$
7. $E(B_j) = \frac{\sum_{i=1}^{m} u_{ij}}{\sum_{i=1}^{m} u_i}$
8. $u_{ij} = D_{ij}\, u_i$

SAMPLE PROBLEMS

SAMPLE PROBLEMS

A Brief Description

Six sample problems have been constructed to evaluate the model. The data used are arbitrarily collected and have no particular significance. Problems one and two are maximizations of different classification systems (called A and B) and describe identical research programs. These two classification systems are initially considered to be independent of one another. Their interdependence is examined in problem five. Problem three experimentally combines the A and B systems by arbitrarily adding their functional equations. Problem four examines the effect of varying the range width of the $W(C_i)$s on the outcome of problem three. Problem five treats the B classification as an additional constraint equation of problem one, and maximizes this new problem. Problem six examines the effect of a budget reduction on the optimum allocation of resources to the categories. To evaluate the sensitivity of the solution, a sensitivity analysis was conducted on the input parameters of problem one. All problems were solved with the aid of a CDC1604 Computer. A discussion of the results of all problems is contained in the conclusions.

General Discussion for Problems One and Two

Problems one and two maximize the effectiveness of the identical research program by means of two assumed independent classification systems (called A and B). The data used in both problems were arbitrarily collected and are without particular significance. The interrelationship between these two problems is treated in problem five.

Objective functions and constraint equations are developed for both problems and solutions are then obtained by using linear programming techniques. An investigation for alternate solutions, and a sensitivity analysis is conducted on problem one.

The model inputs previously discussed will now be tabulated in matrix format as developed by H. Shaller [5] is:

		$C(A_1)$	$C(A_2)$	....	$C(A_n)$
$K(C_1)$	$W(C_1)$	P_{11}	P_{12}	......	P_{1n}
$K(C_2)$	$W(C_2)$	P_{21}	P_{22}	......	P_{2n}
.	.	.		.	
.	.	.		.	
$K(C_m)$	$W(C_n$	P_{m1}	P_{m2}	.	P_{mn}

Table 1. Data display using a matrix format where X = total budget

Problem one   Maximization of the A classification system

DATA

i	$W(C_i)$ \ $C(A_j)$	.130	.521	.194	.074	.033	.020
1	.060-.090	.1	.7	.1	0	0	.1
2	.130-.156	.1	.3	.3	.1	.1	.1
3.	.024-.054	.2	.3	.4	0	0	.1
4	.010-.012	0	.9	0	0	0	.1
5	.064-.084	0	1.0	0	0	0	0
6	.132-.360	.2	.5	.1	0	.1	.1
7	.012-.012	.5	.1	.1	0	.2	.1
8	.100-.108	.2	.3	.2	.1	.1	.1
9	.240-.340	.1	.6	.2	.1	0	0
10	.100-.128	.1	.6	.2	.1	0	0

For this problem arbitrarily take $X = 895.300$ dollars.

For simplicity $k(C_i)$ has been chosen a constant value of forty thousand dollars.

Table 2. Input Data to Problem One

Objective Function

The total effectiveness of the research program is defínted to be:

$$\bar{E} = \sum_{i=1}^{m} E_{ia} = \sum_{i=1}^{m} u_i E_i' = \sum_{i=1}^{m} \sum_{j=1}^{n} u_i E_{ij} = \sum_{i=1}^{m} \sum_{j=1}^{n} p_{ij} C(A_j) u_i$$

For Problem one, therefore, the objective function to be maximized is:

$$\bar{E}_A = \frac{1}{40} (.399x_1 + .240\, x_2 + .262\, x_3 + .471\, x_4 + .521\, x_5 + .311\, x_6 + .145\, x_7 + .234\, x_8 + .372\, x_9 + .372\, x_{10})$$

where the coefficient for x, is determined from the relationship

$$\sum_{j=1}^{6} p_{ij} C(A_j)$$

Accordingly:

$$(.1)(.130)+(.7)(.521)+(.1)(.194)+(.1)(.020)=.399$$

Constraint Equation Format

The constraint equations are determined from the $W(C_i)$ data. Also

$$x_i = W(C_i)X$$

where $\sum_{i=1}^{10} x_i =$ 895.3 thousand dollars

Accordingly:

C_i	x_i (in thousands of dollars)
1	$53.7 \leq x_1 \leq 80.6$
2	$116.3 \leq x_2 \leq 139.9$
3	$21.5 \leq x_3 \leq 48.4$
4	$8.9 \leq x_4 \leq 10.8$
5	$57.5 \leq x_5 \leq 75.5$
6	$118.2 \leq x_6 \leq 322.3$
7	$10.8 \leq x_7 \leq 10.8$
8	$89.5 \leq x_8 \leq 96.7$
9	$214.9 \leq x_9 \leq 304.4$
10	$89.5 \leq x_{10} \leq 114.6$

The objective function developed for problem one can now be maximized subject to the above linear constraint equations by standard linear programming techniques.

Results of Problem one

Table 3 displays various quantities that may be of interest to the research manager. Column one indicates the support a category should receive for maximum effectiveness in the research program. The total number of research units contributed by the ith category to the program, U_i, is obtained by dividing x_i by the cost of a research unit of the ith category, $K(C_i)$. U_i may be broken down into the number of research units which correspond to a particular attribute by the relationship:

$$u_{ij} = p_{ij} \quad U_i$$

Therefore:

i	x_i	U_i	u_{ij} A_1	A_2	A_3	A_4	A_5	A_6
1	80.6	2.02	.20	1.41	.20	0.0	0.0	.20
2	116.3	2.91	.29	.87	.87	.29	.29	.29
3	21.5	.54	.11	.16	.23	0.00	0.00	.05
4	10.8	.27	0.00	.25	0.00	0.00	0.00	.02
5	75.5	1.89	0.00	1.89	0.00	0.00	0.00	0.00
6	118.2	2.96	.59	1.48	.29	0.00	.30	.29
7	10.8	.27	.13	.03	.03	0.00	.05	.03
8	89.5	2.24	.45	.67	.45	.22	.22	.23
9	282.6	7.07	.71	4.24	1.41	.71	0.00	0.00
10	89.5	2.24	.22	1.24	1.45	.22	0.00	0.00

Table 3 - The u_{ij} array distribution of units of research versus dollar input.

Three other quantities may be of interest to the research manager. They are:

a. E_{ij} = Effectiveness of the i^th category relative to the j^th attribute and is defined by

$$E_{ij} = p_{ij} C(A_j)$$

b. E_i' = Effectiveness of the i^th category relative to all the attributes per research unit and is defined by

$$E_i' = \sum_{j=1}^{n} E_{ij}$$

c. E_{ia} = Effectiveness of the i^th category relative to all the attributes and is defined by

$$E_{ia} = u_i E_i'$$

Therefore:

c	E_{ij} A1	A2	A3	A4	A5	A6	E_i'	E_{ia}
1	.013	.364	.019	.000	.000	.002	.399	.806
2	.013	.156	.058	.007	.003	.002	.240	.977
3	.026	.156	.077	.000	.000	.002	.262	.141
4	.000	.468	.000	.000	.000	.002	.471	.127
5	.000	.521	.000	.000	.000	.000	.521	.985
6	.026	.260	.019	.000	.003	.002	.311	.921
7	.065	.052	.019	.000	.006	.002	.145	.039
8	.026	.15	.038	.007	.003	.002	.233	.523
9	.013	.312	.038	.007	.000	.000	.372	2.628
10	.013	.312	.038	.007	.000	.000	.372	.832

Table 4. The effectiveness array versus categories.

Total system effectiveness $= \bar{E} = \sum_{i=1}^{10} E_{ia} = 7.97$ units

This method of displaying data, (H. Shaller [4]) indicates how much a category contributes to a particular attribute. Thus category four has the greatest contribution to attribute two. An increase in emphasis on Attribute two may possibly be accomplished by increased support to category four.

Alternate Solution to Problem one.

An alternate solution, determined by standard Linear programming techniques, was found to exist for problem one.

	Solution I	Solution II	General Solution
x_1	80.6	80.6	80.6
x_2	116.3	116.3	116.3
x_3	21.5	21.5	21.5
x_4	10.8	10.8	10.8
x_5	75.5	75.5	75.5
x_6	118.2	118.2	118.2
x_7	10.8	10.8	10.8
x_8	89.5	89.5	89.5
x_9	282.6	257.5	$\left\{\begin{matrix} 282.6-25.1k \\ 89.5+25.1k \end{matrix}\right\} 0 \le k \le 1$
x_{10}	89.5	114.6	

Table 5. Category allocations which maximize problem one.

As two solutions exist to problem one, any convex combination of these solutions will maximize this problem, as given under column heading of General Solution.

Sensitivity Analysis of Problem One:

A sensitivity analysis was conducted on problem one to determine the extent of dependence of the solution upon the input parameters. Standard Linear programming procedures were used to determine this dependence. The results are as follows:

A. Sensitivity of the coefficients in the objective function.

After completing the linear programming procedures, it was found that the following relationship among the coefficients must exist if an optimal solution is to be maintained.

Equation 1 $\quad E'_1, E'_4, E'_5 \geq E'_9 \geq E'_2, E'_3, E'_6, E'_7, E'_8, E'_{10}$

where: in the sample problem:

$E'_1 = .399$

$E'_2 = .240$

$E'_3 = .260$

$E'_4 = .471$

$E'_5 = .521$

$E'_6 = .311$

$E'_7 = .145$

$E'_8 = .235$

$E'_9 = .372$

$E'_{10} = .372$

Therefore the solution is most sensitive to an increase in E'_{10} and E'_6 and to a decrease in E'_1. As E'_i is defined as

$$E'_i = \sum_{j=1}^{n} E_{ij} = \sum_{j=1}^{n} p_{ij}\, C(A_j)$$

Changes in the (A_j) can now be examined. As was previously noted under Model Inputs, the contribution an attribute is to make to the overall mission will originate from the upper levels of an organization. Changes, therefore, that do not affect equation one will not change the optimality of the solution previously attained. If equation 1 is no longer true, however, the problem must be re-run.

To investigate the reliability of the solution, the research manager would be interested in noting the effect of varying the p_{ij}s upon the solution. An example of this is:

Example

$C(A_j)$	.130	.521	.194	.094	.033	.020
	A_1	A_2	A_3	A_4	A_5	A_6
Original p_{ij}s for category six	.2	.5	.1	0	.1	.1
New p_{ij}'s for category six	.3	.4	0	.1	.1	.1

original $E'_6 = .311$

new

Consequently, optimality is still maintained.

As $E'_{10} = E'_9$ increase $p_{10,2}$ at the expense of any other $p_{10,j}$, would clearly change the optimal solution.

B. Sensitivity of the $W(C_i)$s

Let $W(C_i)$ upper bound be denoted by

$W(C_i)$ lower bound be denoted by

The optimal solution is maintained providing the following relationship exists.

Equation (2) $X - \bar{b}_9 \leq B \leq X - \underline{b}_9$

where $B = \bar{b}_1 + \underline{b}_2 + \underline{b}_3 + \underline{b}_8 + \bar{b}_4 + \bar{b}_5 + \underline{b}_6 + \underline{b}_7 + \underline{b}_{10} = 612.7$

and where X = total budget = \$895,300.

In problem one this relationship is

$$590.9 \leq 612.7 \leq 680.4$$

Therefore; a change in the $W(C_i)$ that leaves equation (2) unchanged will not affect the optimality of the solution.

Example

original $\underline{b}_9 = 214.9$ new $\underline{b}_9 = 254.9$

$\bar{b}_5 = 75.5$ $\bar{b}_5 = 95.5$

$\bar{b}_4 = 10.8$ $\bar{b}_4 = 20.8$

Therefore $B = 642.7$ $X - \underline{b}_9 = 640.4$

B is not less than $X - \underline{b}_9$ and the problem must be rerun.

Problem Two

Problem two is formulated so that the $W(C_i)$s and the

$K(C_i)$'s are the same as that of problem one. A comparison of category emphasis between these two problems can then be made. A new classification system is used which consists of three attributes instead of the seven used in problem one. The p_{ij}s and the $C(B_j)$s will therefore not be identical to those used in the previous problem. The classification system used in problem two will be denoted as the B system.

Therefore: DATA

$C(B_j)$

i	$W(C_i)$	.363	.510	.120
1	.060-.090	.25	.55	.2
2	.130-.156	.4	.3	.3
3	.024-.054	0	0	0
4	.010-.012	0	0	0
5	.064-.084	0	0	0
6	.132-.360	.57	.43	0
7	.012-.012	.5	.5	0
8	.100-.108	1.0	0	0
9	.240-.340	.10	.85	.05
10	.100-.128	.10	.85	.05

$X = 895,300$ dollars

$K(C_i) = 40,000$ dollars

Table 6. Input data to problem two.

Objective function for problem two:

$$\bar{E}_B = \sum_{i=1}^{10} \sum_{j=1}^{3} \frac{x_i}{K_i(C_i)} P_{ij}\, C(B_j)$$

$$= \frac{1}{40} [.395x_1 + .334x_2 + .426\, x_6 + .437\, x_7 + .363_8 x + .477x_9$$

$$+ .477x_{10}]$$

Coefficients for x_3, x_4, x_5, equal zero

System of constraint equations for problem two.

Same as that of problem one.

Results of Problem two.

The results of problem two are displayed in tables seven and eight. Column one in table seven indicates the support a category should receive for maximum effectiveness in the research program, using this particular classification system. U_i is again calculated from the definition:

$$U_i = \frac{x_i}{k(C_i)}$$

Columns four through six indicate the u_{ij}s for the B classification system. Columns seven through twelve indicate the u_{ij} values for the A classification system of problem one, using the x_i values of problem two. An example of the effect of using the x_i values of problem two on the A system is the decrease of u_{11} from .20 to .13 while u_{61} increased from .59 to .72. As expected, changing the level of category support will affect the amount a category

contributes to a particular attribute and to the entire program. Table eight is computed in an identical manner to that of table three in problem one. Columns one through three indicate the E_{ij} of the B system. Columns four and five display E_i, and E_a respectively. Column six is the new E_{ia} of the A system when using the category supports of the B system. An example of the change in E_{ia} of the A system is the decrease in E_{ia} from .806 to .668.

OPTIMIZATION OF "B" CLASSIFICATION SYSTEM

i	U_i	x_i	u_i B_1	B_2	B_3	A_1	A_2	A_3	A_4	A_5	A_6
1	1.34	53.7	.34	.74	.26	.13	.94	.13	0.00	0.00	.13
2	2.91	116.3	1.16	.872	.873	.29	.87	.87	.29	.29	.29
3	.54	21.5	0.00	0.00	0.00	.11	.16	.23	0.00	0.00	.05
4	.23	8.9	0.00	0.00	0.00	0.00	.21	0.00	0.00	0.00	.02
5	1.44	57.5	0.00	0.00	0.00	0.00	1.44	0.00	0.00	0.00	0.00
6	3.58	143.2	2.04	1.54	0.00	.72	1.79	.36	0.00	.36	.36
7	.27	10.8	.13	.14	0.00	.13	.03	.03	0.00	.05	.03
8	2.24	89.5	2.24	0.00	0.00	.45	.67	.45	.22	.22	.23
9	7.61	304.4	.76	5.07	.38	.76	4.57	1.52	.76	0.00	0.00
10	2.24	89.5	.76	1.90	.11	.22	1.34	.45	.22	0.00	0.00

Table 7. The U_{ij} array: distribution of units of research versus dollar input

i	E_{ij} B_1	B_2	B_3	E'_i	E_{ia}	A system E_{ia}
1	.108	.255	.024	.395	.529	.534
2	.145	.153	.036	.334	.972	.698
3	0.00	0.00	0.00	0.00	0.00	.141
4	0.00	0.00	0.00	0.00	0.00	.108
5	0.00	0.00	0.00	0.00	0.00	.750
6	.206	.219	0.00	.426	1.525	1.114
7	.181	.255	0.00	.437	.117	.039
8	.363	0.00	0.00	.363	.813	.523
9	.036	.434	.006	.476	3.622	2.830
10	.036	.434	.006	.476	1.066	.833

Total Effectiveness $\bar{E}_B = \sum_{i=1}^{10} E_{ia}$

$= 8.62$ units

New $\bar{E}_A = 7.57$ units

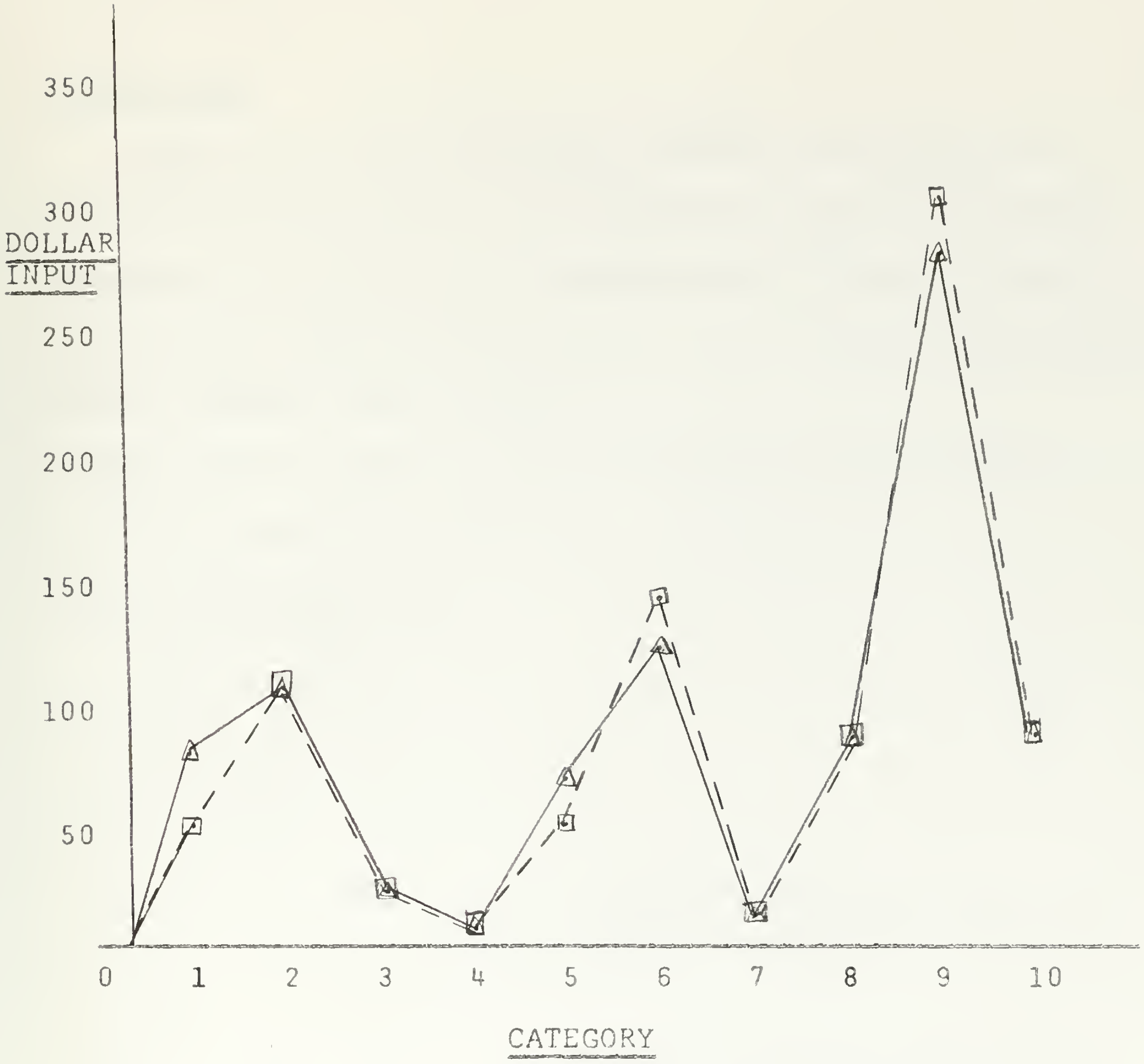
Table 8. The effectiveness matrix versus categories.

Table 9 indicates the results of applying the optimum set of category supports obtained in the A system of problem one to the B system of problem two. Using the x_i of problem one, therefore, we obtain:

		U_{ij}			E_{ij}				
i	x	B_1	B_2	B_3	B_1	B_2	B_3	E_i	E_{ia} (B)
1	80.6	.51	1.11	.40	.108	.255	.024	.395	.798
2	116.3	1.16	1.07	1.07	.145	.153	.036	.334	.972
3	21.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	10.8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	75.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6	118.2	1.68	1.28	0.00	.206	.219	0.00	.426	1.262
7	10.8	.13	.14	0.00	.181	.255	0.00	.437	.118
8	89.5	2.24	0.00	0.00	.363	0.00	0.00	.36	.813
9	282.6	.71	6.01	.35	.036	.434	.060	.476	3.365
10	89.5	.22	1.90	.11	.036	.434	.060	.476	1.066

Table 9. Application of the optimum set of category supports obtained in the A system of problem one to the B system of problem two.

$\tilde{E}_1$ = 8.39 units



Graph 2. Dollar input vs Category: A comparison of the results of the A and B classification systems in problem one and two respectively.

———— △ A = classification system (optimized)

– – – – □ B = classification system (optimized)

Problem III

The A and B classification systems were combined by arbitrarily adding the like components of their functional equations to determine the relationship, if any, between this problem and problems one and two. The resulting problem was to maximize the equation:

$$.748x_1+.574\,x_2+.262\,x_3+.471x_4+.521\,x_5+\,.737\,x_6+.581\,x_7$$
$$+\,.597\,x_8+\;,.849\,x_9+\,.849\,x_{10}$$

Results of Problem III

	x_i	U_i
x_1	53.7	1.34
x_2	116.3	4.07
x_3	21.5	.54
x_4	8.9	.23
x_5	57.5	1.44
x_6	118.2	2.96
x_7	10.8	.27
x_8	89.5	2.24
x_9	304.4	7.61
x_{10}	114.5	2.86

k_i = 40.0 for all x

u_i = Number of research units in the i^{th} category

Total effectiveness: 16.23 units.

Table 10. Results of problem three, maximization of function determined by summing corresponding components of A and B classification systems.

	Dollar Input		Classification Systems		
	Low	High	A	B	A+B
x_1	53.7	80.6	H	L	L
x_2	116.3	139.9	L	L	L
x_3	21.5	48.4	L	L	L
x_4	8.9	10.8	H	L	L
x_5	57.5	75.5	H	L	L
x_6	118.2	322.3	L	143.2	L
x_7	10.8	10.8	S	S	S
x_8	89.5	96.7	L	L	L
x_9	214.9	304.4	282.6	H	H
x_{10}	89.5	114.5	L	L	H

S = same
H = high
L = low

Table 11: A comparison of values which maximized effectiveness of the A, B, and A+B classication systems.

From the results indicated indicated in table 8, no particular relationship can be discerned between problem three and problems one and two.

Problem 4

In some instances a larger degree of uncertainty may exist on the part of the research manager concerning the range of separation between the lower and upper bounds of a category. Therefore, problem four was run to note any effect on the results of problem three by initially choosing a wider range of category emphasis bounds.

Accordingly, the $W(C_i)$s of problem three were varied by increasing the range between the lower and upper bounds

of each category. The lower and upper bounds were lowered and raised respectively by equal amounts for all categories. Maximization of problem three with respect to these new constraint equations was then performed. The bounds were varied in this manner four times, with the results indicated in table 12.

PROBLEM IV

	1		2		3		4	
	LOW	HIGH	LOW	HIGH	LOW	HIGH	LOW	HIGH
1	43.7	90.6	33.7	106.6	13.7	126.6	0.0	146.6
2	106.3	149.9	96.3	159.9	76.3	179.9	56.3	199.9
3	11.5	58.4	0.0	68.4	0.0	88.4	0.0	168.4
4	3.9	21.8	0.0	31.8	0.0	51.8	0.0	71.8
5	47.5	85.5	37.5	95.5	17.5	115.5	0.0	135.5
6	108.2	332.3	98.2	342.3	78.2	362.3	58.2	382.2
7	0.0	20.8	0.0	30.8	0.0	50.8	0.0	70.8
8	79.5	106.7	69.5	116.7	49.5	126.7	29.5	146.7
9	204.9	314.4	194.9	324.4	174.9	344.4	154.9	364.4
10	79.5	124.6	69.5	134.6	49.5	154.6	29.5	174.6

Total Effectiveness

Problem

1	2	3	4
16.62	16.97	17.35	17.71

PROBLEM

1	2	3	4
H	H	H	H
L	L	L	L
L	L	L	L
L	L	L	L
L	L	L	L
117.0	126.4	126.4	123.9
L	L	L	L
L	L	L	L
H	H	H	H
H	H	H	L

Table 12: Comparison of value choices of four runs after varying $W(C_i)$ intervals.

Problem 5.

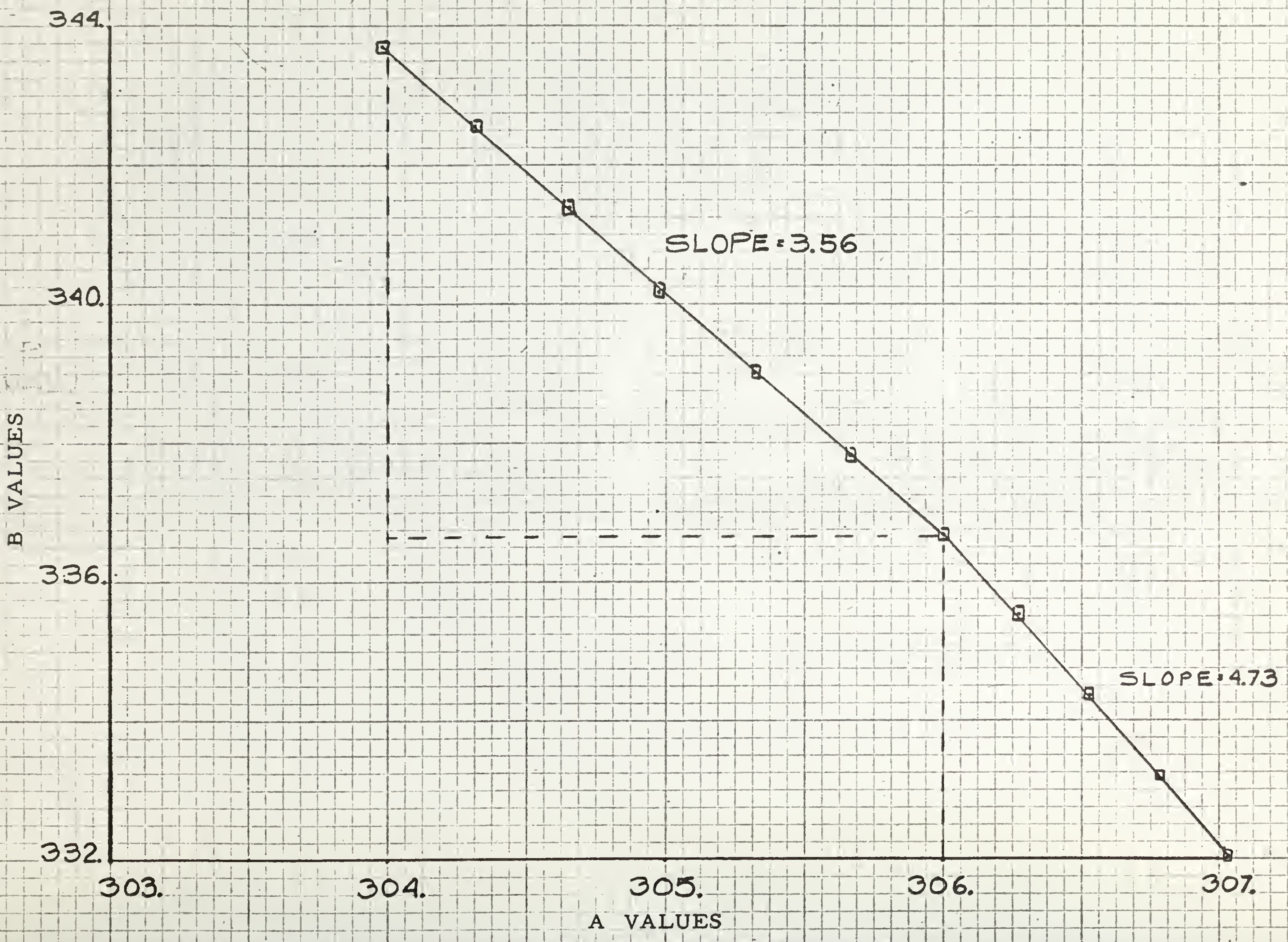
Since the research manager is usually confronted with a situation in which his categories simultaneously emphasize attributes of different classification systems, the model is more realistic when classification systems are coupled. Problem 5 was run to determine what relationship exists when two classification systems, A and B, of problems one and two, respectively, are combined.

Therefore, a solution was obtained to the problem of maximizing the A classification system's objective function using the object function of the B system as an additional constraint equation. Values chosen for the B system function, while acting as a constraint equation for the A system, ranged from its maximum value of 343.69 (obtained from problem two) to 332.0. The lower value, 332.0, was determined by evaluating the B system function with respect to those x values which had maximized the A system. The results of problem five are displayed in table 13 and graph 3.

x Output

Part	Max A	B	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
1	307.016	332.0	78.5	116.3	21.5	8.9	73.9	118.2	10.8	89.5	286.5	89.5
2	306.763	333.2	78.5				73.9				286.5	89.5
3	306.531	334.3	69.5				73.9				295.1	89.5
4	306.275	335.5	53.7				73.9				304.4	94.6
5	305.999	336.7	53.7				73.2				304.4	97.17
6	305.680	337.8	53.7				71.0				304.4	99.48
7	305.330	339.0	53.7				68.5				304.4	102.0
8	304.985	340.2	53.7				66.0				304.4	104.5
9	304.637	341.4	53.7				63.6				304.4	107.0
10	304.318	342.5	53.7				61.3				304.4	109.3
11	303.973	343.69	53.7				58.9				304.4	111.8
x Input	Low		53.7	116.3	21.5	8.9	57.5	118.2	10.8	89.5	214.9	89.5
	High		80.6	139.9	48.4	10.8	75.5	322.3	10.8	96.7	304.9	114.6

Table 13 Maximum effectiveness of the "A" system subject to values of "B" system constraint equations



GRAPH 3 Maximum Effectiveness Comparison Table

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Problem 6

Problem six investigates the effects of a budget cut, a problem sometimes encountered by the research manager. After maximizing problem five, part five, the total budget was cut from $895,300 to $825,300. The effects of the budget decrease on the optimal solution of the problem are displayed in table 14. Care was taken in choosing the new total budget to remain above the sum of the lower budget limits of all categories.

Original budget = $895,300 New budget = $825,300

	x_i values Low.	x_i values High	Optimum category emphasis		revised optimum category emphasis	
1	53.7	80.6	53.7		53.7	
2	116.3	139.9	116.3		116.3	
3	21.5	49.4	21.5		21.5	
4	8.9	10.8	18.9		8.9	
5	57.5	75.5	73.2	Maximum Effectiveness = 7.4 units	57.5	Maximum Effect-iveness = 6.2 units
6	118.2	322.3	118.2		118.2	
7	10.8	10.8	10.8		10.8	
8	89.5	96.7	89.5		89.5	
9	214.9	304.4	304.4		25.94	
10	89.5	114.5	97.17		89.5	

Table 14. Comparison of results of category emphasis before and after budget cut.

CONCLUSIONS

CONCLUSIONS

In the introduction, various questions were raised that were of interest to the research manager. An examination of the results of the six problems and the sensitivity analysis enables us to answer these questions.

Results of problem one reveal several facts that are in agreement with intuition. After examination of the E_1^y column, (page 34), emphasis would intuitively be placed on C5, C4, C1, C9, and the remaining categories in order of their descending amounts. The intuitive allocation method would emphasize those categories which contributed the greatest amount to the research program, per unit of research. The upper values of these categories should always be chosen in descending order of their contribution until a point is reached in the allocation process that necessitates a shift to the lower bounds of the remaining categories. This emphasis shift would, of course, be necessary in order for the program to remain within the total budget. The results of the linear programming technique on problem one were in agreement with those derived from the above intuitive considerations.

Problem one has an alternate solution. However, this situation cannot always be expected. An interpretation of this situation is that the research manager has an alternative

program he may follow and still achieve optimum results. In this problem, resources have been shifted from C9 to C10, resulting in maximum emphasis on this category. Actually, any convex combination of these two solution sets would also be optimal. An alternate solution would also be advantageous in possibly increasing the effectiveness of other classification systems. Thus, if greater emphasis is desired placed upon an attribute of another classification system no loss in program effectiveness need occur. A shift to the alternate solution might increase the emphasis on this attribute while still maintaining the same maximum effectiveness as before.

The reliability of any output is determined to a great extent by the sensitivity of the results on the input parameters. In the sensitivity analysis conducted on problem one, the results (page 36) indicate that category nine is of major importance. Examination of these data indicates that the solution of the problem is most sensitive to changes in categories 10, 6, and 1, in that order, and fairly insensitive to changes in the other coefficients. With this information, emphasis changes in the $C(A_j)$ can now be analyzed.

As each cost coefficient is derived from the equation $E_i = \sum_{j=1}^{n} p_{ij} C(A_j)$ uncertainty in the exact value of p_{ij} may affect the ordering relationship above and thus affect the solution. Assuming our values of p_{ij} are reliable, a change

in the $C(A_j)$ that does not affect the ordering relationship of the categories, will not change the existing program.

Examining the analysis of the $W(C_i)$s, several interesting facts emerge. All categories except category nine have one bound that has no effect on the solution. The lower bound on category one and the upper bound on category two are examples of this. Category nine is the exception, as is indicated by the results. If a shift in category emphasis is however desired, the quantity B is the indicator on how to proceed. Thus, if it is desired to place more emphasis on category one, two alternatives are available to maintain optimality. One alternative would be to still maintain the relationship $X - \bar{b}_9 \leq B \leq X - \underline{b}_9$. The second alternative would be to shift resources among the variables of which B is a function. Thus, resources could be taken from category two, or a combination of categories to accomplish this emphasis change. This information would, of course, also assist the research manager in determining what values to perturb in investigation of other solutions.

Problem two also satisfies our intuition with respect to the E_i'. Here E_9' has the greatest value of all the categories. Once again, intuition would indicate that the greatest emphasis should be placed on category nine. After considering the total budget, a change of emphasis to the remaining category's lower bounds would then take place.

After solving the problem by linear programming techniques, the solution was found to agree with that of the intuitive approach.

Whether this method can be generally employed is a question for further investigation. If the intuitive approach proves correct, linear programming techniques need not be used to maximize individual classification systems. As the intuitive approach was unsuccessful in predicating results for coupled classification systems as in problem five, linear programming techniques may be highly useful in these situations.

The maximization of the B system results in a loss of (0.40) units of research as measured in the A system. A comparison of these two systems in graph two can be misleading. Graph two indicates that to change the emphasis from the A to the B system, categories 1, 5, 6, and 9 should be perturbed. This is not the case, however, as problem 5 indicates. A problem also arises in finding the best position between these two extremes and determining the extent to which the categories should be re-emphasized. This information is also revealed in problem five.

The summing of the A and B system would be of interest if such a relationship had a particular meaning to a research organization. It was done here for experimental

purposes. As a new function was created by this operation, it is reasonable to suppose that the resulting set of x's which maximize this problem could not be predicted. Such was the case.

Changes to the $W(C_i)$'s were accomplished and the results are displayed in table 12. These results are in accordance with theory in that no significant change in the x values chosen occurs. For categories 1, 3, 4, 5 and 7, the lower bounds were eventually placed at zero to note any effect upon the results. No change in category emphasis occurred. Placing a lower bound of a category at zero can assist the research manager in deciding whether to support a particular category. This situation may arise when a new category is to be added to an existing program or when a question of continuing support to a category is raised. If, for example, the set of $W(C_i)$ in Problem 4, Part 4, had been chosen, categories 3, 4, 5, and 7 would not have been supported while category one would have received support.

In Problem 5, the A classification system was optimized with respect to the B classification system for various B values. This problem can aid the research manager in determining to what degree to vary a category and how this variance will change effectiveness of a particular system.

Thus, a specific guide concerning the method of increasing emphasis on attributes of other classification systems is at hand. Table 13 would indicate the optimal manner by which resources should be allocated to accomplish this. A directive to increase emphasis on a particular attribute can now be optimally accomplished using a specified category reemphasis.

Graph 3, the maximum effectiveness comparison between the A and B classification systems, is non-linear. Specifically, a loss of one unit of effectiveness from 307 to 306 units in the A system results in a net gain to the B system of 4.7 units. A decrease of the A system from 306 to 305 units results in a lower gain of 3.5 units. A research manager might feel that a loss of two units in the A system was worth the increase of eight units in the B system. As this problem represents reality to a greater degree than do the others, it is the most important.

The point at which the A system equals 306 units is of interest. This point is the intersection between the two lines of slope 3.56 and 4.73. A point of changing slope on the graph may result due to the particular configuation of the objective function that is maximized. Further study is needed to explain this phenomenon.

Problem 6 reveals the results of a cut in the budget. The results indicate that all categories are placed at their lower limit with the exception of category nine. This method

of adjustment will not always be the case but will depend on the amount that the budget is cut. Category 9 received the remainder of the funds available after all other categories had been supported to their lower limit. As Category 9 has the highest E_i'', this reapportining method is in agreement with intuition.

To evaluate the effect upon the system by the addition of a new category, the problem can be altered to fit these new conditions and the program re-run.

In the hands of a skilled research manager, the Linear programming technique can be a valuable aid. By recognizing the limitations of the model and by applying his experience, the research manager can gain information on the workings of his program. If a "yardstick" is available to measure research output in order to bring greater accuracy to the p_{ij}s, then this technique will be of great interest.

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APPENDIX I

APPENDIX I

GLOSSARY [1]

Attribute - A reason for support of a category and a working level translation of a portion of the mission (objective) of ONR. Hopefully, it is quantifiable.

Balanced Program - That program which includes an appropriate amount of effort in every field of service in which we may reasonably be expected to engage is a balanced program. The operative factors are therefore the list of services required by the ONR Mission (i.e., Objectives) and the proportions of the total effort to be devoted to each.

Boundary Conditions - The constraints on allocation of resources to categories.

Category - A research effort composed of a set of tasks which have a common center of interest. The division is one which should provide a unit convenient for planning purposes, rather than being related to description of the total program as in the case of "projects" and "sub-projects."

Mission - The job imposed by orders, instructions, directives, and the like. In using the term, it should be made clear what group is being referred to, as "Navy Mission," etc. The ONR Mission is defined in complete detail by ONR Instruction 5430.1B of 16 August 1961.

[1] Ad Hoc Research Planning Committee at the Office of Naval Research

Objectives - A more detailed description of the ONR Mission. An objective is thus one of the services, the sum of which constitute the ONR Mission.

Program - Used without an adjective, the sum total of the efforts directed toward accomplishment of the ONR Mission. The Research Program is that portion of the effort which is under the cognizance of the Assistant Chief for Research.

Project - The long-range effort of a developing agency which extends over the full time span of the development of a system, or that which constitutes classes of work that continue indefinitely. Each project appears as a line item in the Annual Navy RDT&E Program. (Definition required by DOD Directive 5200.10, 1 August 1962). See also "category", "subproject".

Requirement - A plan or statement indicating the need or demand for personnel, equipment, supplies, resources, facilities or services by specific quantities, for specific periods of time or at a specific time (JCS Pub. 1).

Research Planning - The process of defining courses of action to be employed in achieving an effective and balanced research program and the communication of this process to other interested organizations. The selection of courses of action through a systematic consideration of alternatives.

Technological Barrier - An area of ignorance that prevents the development of a desired end item or capability.

APPENDIX II

APPENDIX II

DETAILED PROBLEM ONE FORMULATION

Appendix two consists of the intermediate steps involved in solving problem one. They are provided to assist the reader in reproducing these same results. The notation used may be found in Garvin [1].

FORMULATION FOR COMPUTATION BY SIMPLEX METHOD OF PROBLEM I

Maximize $[1 \ (.399x + .240x + .262x + .471x + .521x$

$+ .311x + .145x + .234x + .372x + .372x)]$

SUBJECT TO

1. $x_1 - x_{11} = 53.7$
2. $x_1 + x_{12} = 80.6$
3. $x_2 - x_{13} = 116.3$
4. $x_2 + x_{14} = 139.9$
5. $x_3 - x_{15} = 21.5$
6. $x_3 + x_{16} = 48.4$
7. $x_4 - x_{17} = 8.9$
8. $x_4 + x_{18} = 10.8$
9. $x_5 - x_{19} = 57.5$
10. $x_5 + x_{20} = 75.5$
11. $x_6 - x_{21} = 118.2$
12. $x_6 + x_{22} = 322.3$
13. $x_7 - x_{23} = 10.8$
14. $x_7 + x_{24} = 10.8$
15. $x_8 - x_{25} = 89.5$
16. $x_8 + x_{26} = 96.7$
17. $x_9 - x_{27} = 214.9$
18. $x_9 + x_{28} = 304.4$
19. $x_{10} - x_{29} = 89.5$
20. $x_{10} + x_{30} = 114.6$
21. $\sum_{i=1}^{10} x_i + x_{31} = 895.3$

where x_{11} thru x_{31} are slack variables

	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	P_{20}	P_{21}	P_{22}	P_{23}	P_{24}	P_{25}	P_{26}	P_{27}	P_{28}	P_{29}	P_{30}	P_{31}
P_1	80.6	1.											1.																			
P_{11}	26.9											1.	1.																			
P_2	116.3		1											-1.																		
P_{14}	23.6													1.	1.																	
P_3	21.5			1.												-1.																
P_{16}	26.9															1.	1.															
P_4	10.8				1														1.													
P_{17}	1.9																	1.	1.													
P_5	75.5					1.															1.											
P_{19}	18.0																			1.	1.											
P_6	118.2						1.															-1.										
P_{32}	204.1																					1.	1.									
P_7	10.8							1.																-1.								
P_{24}	0.																							1.	1.							
P_8	89.5								1.																	-1.						
P_{26}	7.2																									1.	1.					
P_9	232.6									1.			-1.	1.	1.	1.			-1.		-1.	1.		1.		1.				1.		1.
P_{28}	21.8												1.	-1.	-1.	-1.			1.		1.	-1.		-1.		-1.			1.	-1.		-1.
P_{10}	89.5										1.																			-1.		
P_{30}	25.1																													1.	1.	
P_{27}	67.7												-1.	1.	1.	1.			-1.		-1.	1.		1.		1.		1.		1.		1.
Z_j		-.399	-.240	-.262	-.471	-.521	-.311	-.145	-.234	-.372	-.372	0.	-.027	-.132	0.	-.11	0.	0.	-.099	0.	-.199	-.061	0.	-.227	0.	-.133	0.	0.	0.	0	0	-.372
C_j		-.399	-.240	-.262	-.471	-.521	-.311	-.145	-.234	-.372	-.372	0.	0.	0	0.	0.	0.	0.	0.	0.	0.	0.	0	0.	0.	0.	0.	0.	0.	0.	0.	0.

TABLE 15 FINAL TABLEAU OF OPTIMAL SOLUTION OF PROBLEM ONE

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-1	1																			
0		1																		
		-1	1																	
				1																
				-1	1															
							1													
						-1	1													
								-1	1											
										1										
										-1	1									
												1								
												-1	1							
														1						
														-1	1					
	-1	-1	-1				-1		-1	-1		-1		-1				-1		1
	1	1	1				1		1	1		1		1			1	1		-1
																		1		
																		-1	1	
0	-1	-1	-1				-1		-1	-1		-1		-1		-		-1		1

Table 16. Inverse of optimal basis of problem one.

		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-1	1																			
		1																		
		-1	1																	
				1																
				-1	1															
							1													
						-1	1													
									1											
								-1	1											
										1										
										-1	1									
												1								
												-1	1							
														1						
														-1	1					
	-1	-1		-1			-1		-1	-1		-1		-1				0		
	1	1		1			1		1	1		1		1			1	0	-1	1
																		0	1	-1
																		-1		
	-1	-1		-1			-1		-1	-1		-1		-1		-1		0		1

Table 17. Inverse of alternate optimal basis of problem one.

APPENDIX III

APPENDIX III

Examples of Attributes, Branches, Categories and Classification System reprinted from the Ad Hoc Research Planning Committee Report.

List of Some Existing Attributes - Not Independent

1. Scientific value
2. Military value
3. Technological value
4. Windows
5. Dissemination
6. Coordination of Research
7. Prestige
8. Contract with Scientific Community
9. Pressures and interests
10. Existance of adequate programs within other Government agencies
11. Industrial incentive

Partial List of ONR Branches

1. Acoustics Branch
2. Georgraphy Branch
3. Geophysics Branch
4. Field Projects Branch
5. Metallurgy Branch
6. Chemistry Branch

7. Propulsion Chemistry Branch
8. Power Branch
9. Physics Branch
10. Nuclear Physics Branch
11. Electronics Branch
12. Mathematics Branch
13. Logistics and Mathematical Statistics Branch
14. Information Systems Branch
15. Fluid Dynamics Branch
16. Structural Mechanics Branch
17. Physiology Branch
18. Biochemistry Branch
19. Microbiology Branch
20. Medicine and Denistry Branch
21. Biology Branch
22. Group Psychology Branch
23. Physiological Psychology Branch
24. Engineering Psychology Branch
25. Personnel and Training Branch

Breakdown of Electronics Branch into Categories

ELECTRONICS BRANCH

Areas of New Emphasis

a. Submarine Detection
b. Communications
Communications Theory

Current Program Categories

a. Circuit Analysis and Synthesis
b. Information Theory and Coding
c. Data Processing
d. Bio-Electronics
Physical Electronics

Current Program Categories

a. Solid State Electronics
b. Cathode Characteristics (Thermionics, Field Emission)
c. Electron Ballistics
d. Plasma Studies
Electromagnetic Wave Propagation and Radiation

Current Program Categories

a. Anomalous Propagation Modes
b. VLF and ELF
c. Antennas
d. Geomagnetics
e. Direction Finding
Electronic Components

Current Program Categories

Solid State Devices
b. Microelectronics
c. Electron Tubes
d. Application of New Materials
Radio Astrophysics

Current Program Categories

a. Solar Flare Studies
b. Radio Source Positioning
c. Cosmology via Radio Astronomy
d. Planetary Astronomy

Examples of Classification Systems Now In Use

I. Chronological

A. Short Range
B. Intermediate Range
C. Long Range
D. Other or combination

II. ENVIRONMENTAL (Spatial)

A. Aero Space
B. Surface (Water, Land, Amphibious)
C. Underseas (ASW, Mining Submarine)
D. Other or combination

III. CONVENTIONAL SHIPBOARD

A. Engineering
B. Operations
C. Deck
D. Ordnance
E. Aviation
F. Medical/Dental
G. Supply
H. Other or combinations

IV. OFFENSIVE/DEFENSIVE

A. Offensive
B. Defensive
C. Other or Combination

V. FUNCTIONAL OPERATIONAL

A. Surveillance
B. Command control
C. Intelligence
D. Nullification
E. Delivery Platform
F. Logistics

VI. CATEGORIES

A. Earth Sciences
B. Material Sciences
C. Physical Sciences
D. Mathematical Sciences
E. Biological Sciences
F. Psychological Sciences
G. Operations Analysis
H. Other or combinations

VII. FUNCTIONAL BY BROAD MISSION

A. Applied Research
B. Basic Research
C. Development Test & Evaluation
D. Management & Support

VIII. BUDGET ACTIVITY (End Item Categories)

A. Military Sciences
B. Aircraft and Related Equipment
C. Missiles and Related Equipment
D. Astronautics
E. Ships and Small Craft
F. Ordnance Combat Vehicles
G. Other Equipment
H. Program Wide Management Support
I. Military Family Housing

APPENDIX IV

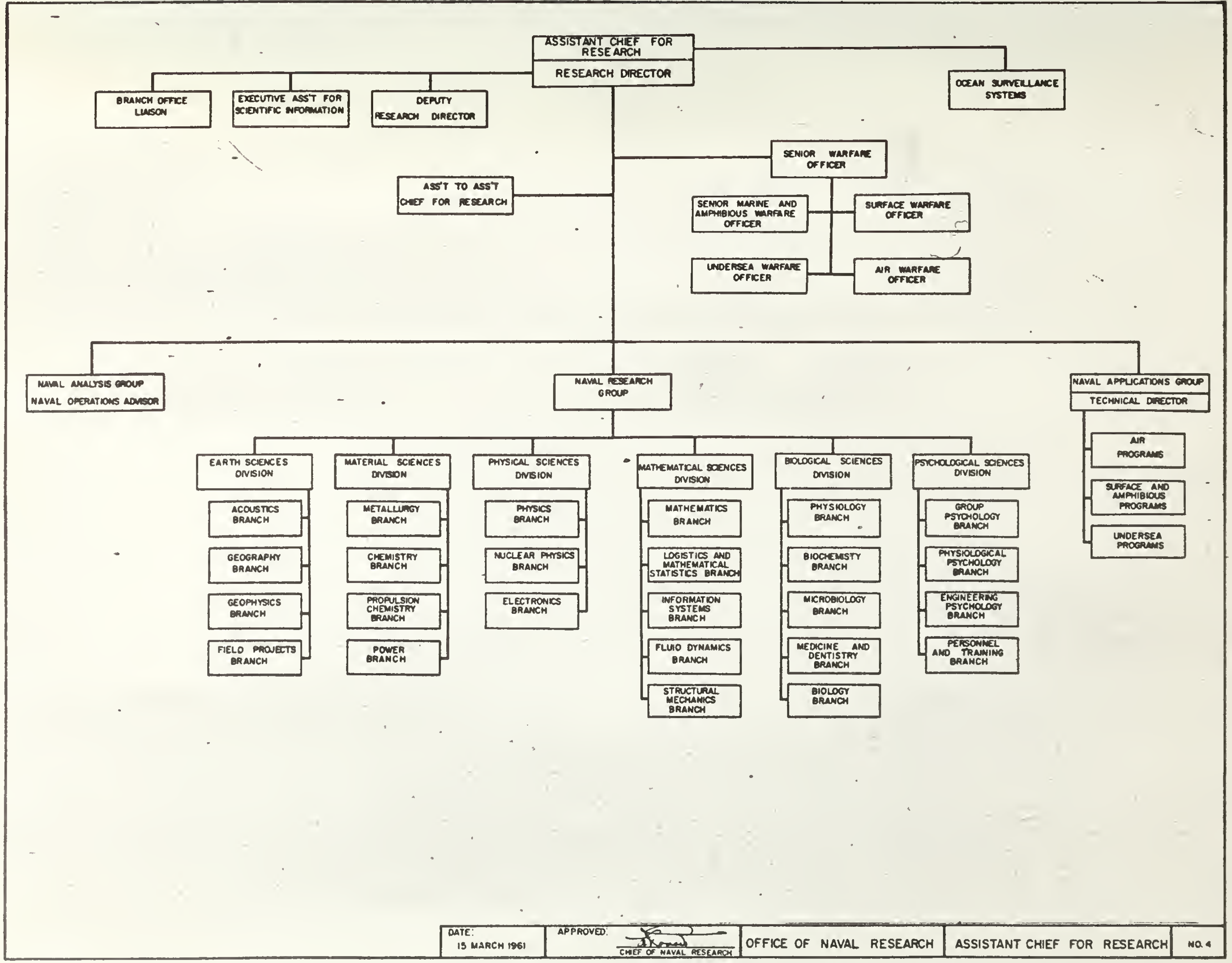

Table 18 Organizational diagram of the Office of Naval Research

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